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Particle−antiparticle description in classical mechanics

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The new approach for the classical mechanics in mordern physical languages is studied in this article. First of all the Newton's laws of motions are restated in the terminology of the quantum mechanics.

Bohr's relation takes a fundamental role in this trial and the concept of energy is the first thing considered. Then the other physical quantities such as the velocity, the momentum, mass, and, etc. are the secondary quantities reduced from the concept of energy.

As for the concept of particle anti−particle the third law of the Newtonian mechanics needs it and the usual action and reaction are related to the concept of particle and anti−particle, respectively.

The part II treats the motion in the phase space or the trajectories in the phase space. If there are the motions of the original particles, then we can associate it the motions of the antiparticles.

I. Introduction

In this article we mainly study the connections between the classical mechanics and the modern physics. The essential part of the classical mechanics is now considered as established branches of physics. A few investigators including the present author think it to be a subject to be more cultivated.

In the elementary course of physical education we have established educational material such as vectorial formalism and so on. Vectorial mechanics itself is introduced into the classical mechanics only in this century. Thus even the Newtonian mechanics has been gradually changed even in its educational forms.

As typical theories of physics we have now three theories which consist of the classical mechanics, the quantum mechanics and the elementary particle physics. The classical mechanics have three laws, namely, the first law of inertia, the second law of the equation of motion and the third law of the action and reaction.

These materials in the classical mechanics consist of the quantities such as the momentum, the velocity, accerelation, mass and force. In the next step the important concept of energy comes into our classical mechanics relating to the first law of thermodynamics.

While, the quantum mechanics is composed mainly from the concept of the energy. Therefore, the Hamiltonian takes a principal role in the quantum mechanics.

II. Part I

From the times of the ancient Greek the concept of motion has been studied by many authors. Among them we can consider the famous Zenon's third argument about motion as related by Aristotle.[1] This Zenon's paradox says that a flying arrow is at each instant locally equivalent to an arrow at rest.

To resolve this paradox we use the concept of infinitesimal or limit. We can define to be at rest as

$$x_1 = \cdot\cdot\cdot\cdot\cdot\cdot = x_2 = \text{const. for a finite interval } [t_1,\ t_2]$$

or we can write as

$$x = f\ (t)\ = \text{const. for } t_1 \leqq t \leqq t_2. \qquad (1)$$

In fact, the motion can be defined as a limit of

$$x_2 \to x_1 \text{ for } t_2 \to t_1 \text{ or } t_2 - t_1 \to 0.$$

The arrow can occupy some space point at the each instant. But the problem is the durating time for both cases. The answer is that the time interval for rest is finite, the infinitesimal small for the motion.

This fact can be written as

$$\lim_{t_2 \to t_1} \frac{\triangle x}{\triangle t} = 0$$

for the case to be at rest.

In the case of motion if the time changes from t_1 to t_2, the space point which the body stays can change from x_1 to x_2. Then we can obtain

$$\lim_{t_2 \to t_1} \frac{x_2 - x_1}{t_2 - t_1} = \lim_{\triangle t \to 0} \frac{\triangle x}{\triangle t} = v \qquad (2)$$

which is the definition of velocity and we use the notations

$$x_2 - x_1 = \triangle x, \qquad \triangle t = t_2 - t_1.$$

The first law of motion or the law of inertia treats these two modes as the unified one and says that

'Every body continues in its state of rest or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.'

From the modern physical point of view we must consider the inertial state to be an eigen state of some observable (an operator) . Here we identify it as one of energy eigen states, i.e.,the stationary state of Bohr's type. Then the constant energy implies to be at rest or the motion with a constant velocity. For the first study we restrict the energy to be the kinetic energy, so the relation

$$E_{kin}=\frac{1}{2}mv^2=E \tag{3}$$

can be used hereafter. As for the potential energy, for examples,

$$U=mgh \quad \text{or} \quad \frac{1}{2}Kx^2,$$

we must consider the field quantities.
The potential energy is not the energy of bodies themselves. But the energy of the environment surrounding them, i. e., spaces. When we treat the mechanics of the material points, we consider only the kinetic energy.

We then think the most fundamental law of the general mechanics to be the following Bohr's type relation

$$\triangle E=E_2-E_1=\text{the energy change of the inside or the outside.}$$

The type of energy of the right hand side is the essential parts of each mechanics and they have different forms.

In the case of quantum mechanics the photon $h\nu$ with the frequency ν takes a role of it and in the Newtonian mechanics the work $\vec{F}\cdot d\vec{s}$ does it.

If we consider the case of classical mechanics the work done can be written down as

$$\triangle E=\int_1^2 \vec{F}\cdot d\vec{s}. \tag{4}$$

Now we may find that the force is one of mechanism of the energy exchange between particles or particles and outside world. Therefore, we can now say that the force is not the essential things, but a convinient concept of energy exchange or interactions between bodies.

The word 'Rikigaku' (in Japanese) is the inappropriate word which leads to misunderstanding. In the word 'Dynamics' and 'Mechanics' there is not the word which means the 'Force'. In mechanics the 'Force' is not 'Primo attore'.
If we can find other mechanism of energy exchange, then we can have the other mechanics of new type which until now is not known.

In the classical case because of appearance of the works or force we must describe the motion in the term of evolution of the momentum

$$\frac{d\vec{P}}{dt}=\vec{F}, \tag{5}$$

this is the famous Newton's law of the motion or the second law of classical mechanics.

The several procedures are needed as follows:

$$\triangle E=\int \vec{F}\cdot d\vec{s} \tag{6}$$

$$\frac{1}{2}mv_2{}^2-\frac{1}{2}mv_1{}^2=\int \vec{F}\cdot d\vec{s} \tag{7}$$

$$d\left(\frac{1}{2}mv^2\right)=Fds \tag{8}$$

$$mv\frac{dv}{dt} = F\frac{ds}{dt} = Fv \tag{9}$$

If $v \neq 0$, $\quad m\frac{d\vec{v}}{dt} = \vec{F}.$ (10)

If we define $\vec{p} = m\vec{v}$,

$$\frac{d\vec{p}}{dt} = \vec{F} \tag{11}$$

which is called the equation of motion and the same as (5).

In the relativistic terminology we rewrite these procedures as

$$\triangle P^i = F^i \triangle t \quad (i = 1,\ 2,\ 3) \tag{12}$$

$$\triangle E = \vec{F} \cdot \triangle \vec{S} = \vec{F} \cdot \frac{\triangle \vec{S}}{\triangle t} = F^4 \triangle t$$

where

$$F^4 = \vec{F} \cdot \vec{v} \tag{13}$$

Lastly as for the third law of the Newtonian mechanics in the appropriate case the absorbtion of energy or emmision of energy corresponds to the action or rea ction, respectively. In other words, if a body absorbs an amount of energy then the other body necessarily loses the same energy for the sake of the conservation of energy. It will be clear that in the case of linkaged bodies A and B, the work done W_{AB} which is the work done to A by B is the opposite of the work W_{BA} which is the work done to B by A, i. e.,

$$W_{BA} = -W_{AB} \tag{14}$$

Then by the condition of linkage

$$d s_A = ds_B$$

we must conclude

$$F_{AB} + F_{BA} = 0. \tag{15}$$

which is the third law.

II. Part II

In this part II we study the particle anti−particle relation in the classical motion especially in the circle. In particular, the Poincarē section is studied.

A Poincarē section is constructed by viewing the phase space diagram stroboscopically in such a way that the motion is observed periodically. The Poincarē method consists of cutting or sectioning the phase space at regular intervals.

As the simplest example we consider the motion of one particle on the unit circle. The particle moves counterclockwise (positive direction) on the circle. Then we view this motion stroboscopically we obtain several points on the circle.

We can also this same picture as the motion of antiparticle which moves clockwise (negative direction) on the circle. The location of the several points is the same as the above particle motion.

The word "particle and antiparticle' is here used as the meaning of the opposite direction. Namely the antiparticle has the opposite sign of velocity. If the figures which some particle figures by its motion are given, then we can interprete it the figures by the antiparticles.

The motions on the circle can have two directions of motion.
Detailed discussion of this motion will be appeared elsewhere with my students. Here we will try to progress further to the motion in the higher dimensional phase space.

In the statistical mechanics we adopt the concept of the trajectories in the phase space. The phase points (P_i) consist of (q_1 , q_2 , $\cdot\cdot\cdot$, p_1 , p_2 ,) and as time passes the trajectories take such a phase points (P_i) successively.

A motion of a point on the phase space is not the real motion but the conceptual motion. If T_t :T_1, T_2, $\cdot\cdot\cdot$ are the particle motions then T_{-1} : T_{-1},T_{-2}, $\cdot\cdot\cdot$ are the anti particle's motion .

For a well-used motion on the Torus, if the motions are periodic we assume the motion of particles as antiparticle motions which are all reverse of the original motions. In details, if the periodic points are P_0, P_1, $\cdot\cdot\cdot P_{n+1}=P_0$, the original particles take the points P_0, P_1, $\cdot\cdot\cdot$. Then the antiparticles take the points P_0, P_n, P_{n-1}, $\cdot\cdot\cdot\cdot\cdot\cdot$, P_1 i. e., the following is true.

(The motion of the original motion) − (the motion of the antiparticle) = 0

The trajectories in phase space accompany the famous Zermero's paradox. In this particle antiparticle description we can rewrite Loschmit's paradox.
This paradox restates as the following words.

Loschmidt's Umkehreinwand:
If the original motion in phase space P_1, P_2, $\cdot\cdot\cdot\cdot\cdot P_n$, $\cdot\cdot\cdot$exist, then the inverse motion of antiparticle in phase space P_n, $\cdot\cdot\cdot\cdot P_2$, P_1, can also exist.

The above statement means that the time reversal is reconsidered as the motion of the antiparticles. These detailed discussions also appear in other papers.

Reference

1) B. Mashhoon, Phys. Lett. A 145 (1990) 147.

古典力学における粒子ー反粒子的記述

（鯖 田 秀 樹）

この論文では古典力学に対する現代物理学の新しい研究方法が研究されている。第1にニュートンの運動の法則が量子力学のことばで再記述される。この試みでボーアの関係が根本的役割をはたし，エネルギー概念が最初に考察されるべきことがらとなる。

ついで，速度，運動量，質量他はこのエネルギー概念から派生する第2義的量であることを示す。粒子ー反粒子概念についてはニュートン力学の第3法則がそれを必要とし，通常の作用ー反作用がそれぞれ粒子，反粒子の概念に結びつく。第2部は位相空間での運動や位相空間内のトラジェクトリーをとりあつかう。あるもとの粒子の運動があれば，その運動に反粒子の運動を付随させられる。