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Spectral Narrowing of Selective Reflection Signal from Rb Vapor

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The selective reflection spectra of Rb D_1 line from a glass vapor interface has been studied as a function of incidence angles of light. It has been observed that the Doppler broadening of the spectra has been partially reduced for the nearly normal incidence of light to the interface, and that the ^{85}Rb hyperfine structure of the excited state $^2P_{1/2}$, which is usually masked in the Doppler width, has been almost resolved as a result of the spectral narrowing. The observed line shapes are in good agreement with the theoretical ones including the effect of atom-wall collision.

I Introduction

The reflected light from a glass-vapor interface exhibits resonant changes near resonance lines of the vapor. This phenomenon is known as the selective reflection, and usually described by the dispersion theory of light [1]. According to the theory, the spectral width $\Delta\nu$ of the selective reflection signal is roughly given by a sum of the Doppler width $\Delta\nu_D$ and the homogeneous width $\Delta\nu_h$ where $\Delta\nu_h = \Delta\nu_n + \Delta\nu_c$, and $\Delta\nu_n$ and $\Delta\nu_c$ are the widths due to the spontaneous emission of the atom and the atom-atom collision, respectively. However, Cojan found experimental evidence on the spectral narrowing from Hg vapor, that is, the spectral width decreases below $\Delta\nu_D$ when $\Delta\nu_D > \Delta\nu_h$ [2]. To account for the experimental results, he proposed a modified theory including the atom-wall collision on which the induced polarizations of colliding atoms are preserved. Later, Woerdman et al. have studied the dependences of Na selective reflection spectrum on the vapor density and reflection angles and clearly observed the spectral narrowing of the signal for nearly normal reflection angles [3]. To analyze their experimental results, Schuurmans has developed the modified theory for the normally reflected light from an atom-vapor interface, taking into account of the diffuse atom-wall collision on which the polarizations of atoms are completely lost [4]. Further, Burgmans et al. have carefully reexamined the experiments of Woerdman et al. and well verified the predictions of the modified theory by Schuurmans [5, 6], although the unresolved hyperfine splittings of the excited states of Na has been left as a source of exta line broadening.

In addition to Na vapor, Rb vapor can be considered to be suitable for a sensible indicator of the spectral narrowing of the selective reflection since the hyperfine splittings in the excited state $^2P_{1/2}$ are comparable to the Doppler width.

In this paper, we report the first observation of the dependence of the narrowing of Rb D_1 line on reflection angles. According to the modified theory [4], we have calculated the line shapes of the selective reflection spectra for various reflection angles, which have not been treated in the case of Na vapor. We have observed the resolution of the hyperfine splitting in the state $^2P_{1/2}$ of ^{85}Rb as a result of the spectral narrowing. The experimental results are compared with the theoretical predictions.

II Theory

As shown in Fig. 1, we assume that there is a glass-vapor interface in the y-z plane and that the electric field of light $E_i(r, t)=E_i \exp[-i(\omega t-k_i \cdot r)]$, σ polarized in the z-direction, is obliquely incident on the interface and reflected back from the interface as $E_r(r, t)=E_r \exp[-i(\omega t-k_r \cdot r)]$. The electric field in the vapor is indicated by $E(r)\exp(-i\omega t)$ in the z-direction, which reduces to $E_o(r, t)=E_o(r)\exp(-i\omega t)$ in the absence of the vapor. Considering the continuity of the electric and magnetic fields across the interface, we can derive the boundary conditions

$$E_i+E_r=E(0)$$
$$ink\cos\phi(E_i-E_r)=\frac{\partial E(0)}{\partial x} \tag{1}$$

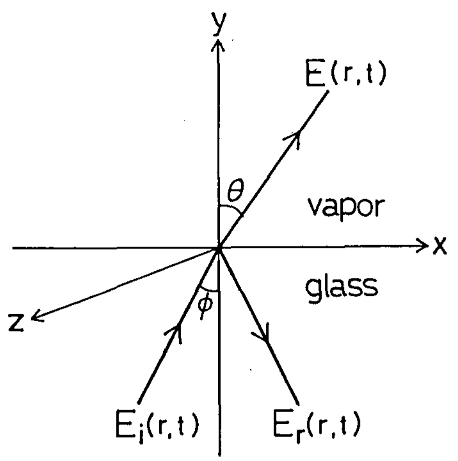

Fig. 1 : Schematic diagram of the electric fields near a glass-vapor interface.

where k and n are the wave number of light in the vacuum and the refractive index of glass, respectively. The polarization $p(r, t)$ of an atom with resonance frequency ν_o, which is induced in the z-direction by $E_o(r, t)$, satisfies

$$\frac{d^2p}{dt^2}+2\gamma\frac{dp}{dt}+(\gamma^2+\omega^2)p=\frac{e^2f}{m}E_o(r)\exp(-i\omega t) \tag{2}$$

where $\omega_o=2\pi\nu_o$, γ the damping costant of the atom, f the oscillator strength, e the electron charge and m the electron mass. Since the condition $|\omega-\omega_o|<<\omega_o$ is satisfied in our experimental situation, Eq. (2) reduces to

$$\frac{dp}{dt}+(\gamma+i\omega_o)p=\frac{ie^2f}{2m\omega_o}E_o(r)\exp(-i\omega t) \tag{3}$$

The polarization $p(r, t; v_r)$ of the atom with velocity component v_r along the propagation direction of the electric field in the vapor satisfies

$$\frac{d}{dt}p(r, t; v_r)=\frac{\partial}{\partial t}p(r, t; v_r)+v_r\frac{\partial}{\partial r}p(r, t; v_r). \quad (4)$$

Putting $p(r, t; v_r)\exp(-i\omega t)$, we have the following relation for $p(r; v_r)$

$$\frac{d}{dt}p(r; v_r)+\frac{\gamma+i\Delta\omega}{v_r}p(r; v_r)=\frac{ie^2 f}{2m\omega_o v_r}E_o(r) \quad (5)$$

where $\Delta\omega=\omega_o-\omega$.

We put $p(r; u, v)=p(r; v_r)$ for $v_r=u\cos\theta+v\sin\theta$ where u and v are the x-and y-components of atomic velocity, respectively, and introduce a small positive number ε into $E_o \exp(ikr)$ such that it converges for $r\to\infty$ [4]. Consider first atoms that move towards the interface, having $u<0$. Then, the boundary condition $p(r; u, v)\to 0$ for $r\to\infty$ yields:

$$p(r, u<0, v)=\frac{ie^2 fE_o}{2m\omega_o}\frac{1}{\gamma+i(\Delta\omega+kv_r)}\exp[(i-\varepsilon)kr]. \quad (6)$$

If the atoms collides diffusely with the interface, we find

$$p(r, u<0, v)=\frac{ie^2 fE_o}{2m\omega_o}\frac{1}{r+i(\Delta\omega+kv_r)}\left\{\exp[(i-\omega)kr]-\exp\left[-\frac{\gamma+i\Delta\omega}{v_r}r\right]\right\}. \quad (7)$$

Since u and v are statistically independent, the macroscopic polarization $P(r)$ of the vapor is given by the relation

$$P(r)=\rho\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}p(\mathrm{r}; u, v)V(u)V(v)\mathrm{d}u\mathrm{d}v \quad (8)$$

where ρ is the density of vapor, and $V(u)$ and $V(v)$ are the Maxwell distributions at absolute temperature T of the vapor

$$V(u)=\sqrt{\frac{m_A}{2\pi k_B T}}\exp\left(-u^2\frac{m_A}{2k_B T}\right) \quad (9)$$

and so on, where m_A is the mass of atom and k_B the Boltzmann constant.

The electric field $E(r)$ in the vapor is given by

$$\mathrm{E}(r)=E_o(r)+E_1(r)=E_0(r)+e_1(r)E_o(r) \quad (10)$$

where $E_1(r)$ is the contribution from the macroscopic polarization $P(r)$ induced by $E_o(r)$. The electric field $E(r)$ satisfies the Maxwell equation in the presence of $P(r)$

$$\frac{d^2}{dr^2}E(r)+k^2(1+2i\varepsilon)E(r)=\frac{k^2}{\varepsilon_o}P(r) \quad (11)$$

which reduces to

$$\frac{d^2}{dr^2}e_1(r)+2(i-\varepsilon)k\frac{d}{dr}e_1(r)=-\frac{k^2}{\varepsilon_o E_o(r)}P(r) \quad (12)$$

The boundary condition $\mathrm{d}e_1(r)/\mathrm{d}r\to 0$ for $r\to\infty$ yields a solution of Eq. (12)

$$\frac{d}{dr}e_1(o)=\int_0^{\infty}\frac{k^2}{\varepsilon_0 E_0}P(r)\exp[(i-\varepsilon)kr]dr \tag{13}$$

The reflectivdty R of the interface is given by

$$R=\left|\frac{E_r}{E_i}\right|^2 \tag{14}$$

which can be written as

$$R=\left|\frac{n\cos\phi-\dfrac{\cos\theta}{ikE(0)}\dfrac{\partial}{\partial r}E(0)}{n\cos\phi+\dfrac{\cos\theta}{ikE(0)}\dfrac{\partial}{\partial r}E(0)}\right|^2 \tag{15}$$

The selective reflection signal $S(\omega_o-\omega)$ of the vapor is given by the extra reflectivity R-R_o where R_o is the reflectivity of a glass-vacuum interface as given by

$$R_o=\left|\frac{n\cos\phi-\cos\theta}{n\cos\phi+\cos\theta}\right|^2. \tag{16}$$

Using Eqs. (8), (13) and (15), the selective reflection signal $S(\omega_o-\omega)$ can be represented in a convenient form for the calculation as shown in the following

$$S(\omega_o-\omega)=R-R_o=-\frac{e^2 f\rho R_o}{m\pi\varepsilon_o\omega_o\gamma_D}\frac{2n\cos\phi}{n^2\cos^2\phi-\cos^2\theta}\times$$
$$\times\int_{-\infty}^{\infty}\int_{\infty}^{o}\frac{[\Delta\omega+(\tilde{u}\cos\theta+\tilde{v}\sin\theta))]\exp(-\tilde{u}^2)\exp(-\tilde{v}^2)d\tilde{u}d\tilde{v}}{(\ln 2)\Gamma^2+[\tilde{\Delta}\omega+(\tilde{u}\cos\theta+\tilde{v}\sin\theta)]^2} \tag{17}$$

Here we have introduced $\tilde{u}=u/u_m$, $\tilde{v}=v/v_m$ and $\widetilde{\Delta w}=\Delta\omega/\gamma_D$ where $u_m=v_m=\sqrt{2k_B T/m_A}$, $\Delta\omega=\omega_o-\omega$, $\gamma_D=ku_m$ and $\Gamma=(\Delta\nu_n+\Delta\nu_C)/\Delta\nu_D$.

Figure 2 shows the energy level structure of D_1 line for ^{85}Rb and ^{87}Rb atoms. Allowed transitions are indicated by T_i for ^{85}Rb and T_i' for ^{87}Rb together with their relative transition probabilities. The hyperfine splitting ΔE is denoted by the relative value to the fine structure level. We assume that transitions T_i and T_i' contribute to the selective reflection spectrum of vapor as $S_i(\omega_{oi}-\omega)$ and $S_i'(\omega_{oi}'-\omega)$, respectively, which are given by Eq. (17), where ω_{oi} is the resonance frequency of T_i and ω'_{oi} that of T_i'. Then, the selective reflection spectrum of Rb vapor is given by

$$R-R_o=\sum_i[a_i S_i(\omega_{oi}-\omega)+b_i S_i'(\omega'_{oi}-\omega)] \tag{18}$$

where a_i and b_i are to be determined from the transition probabilities of T_i and T_i' and the abundance ratio of Rb vapor.

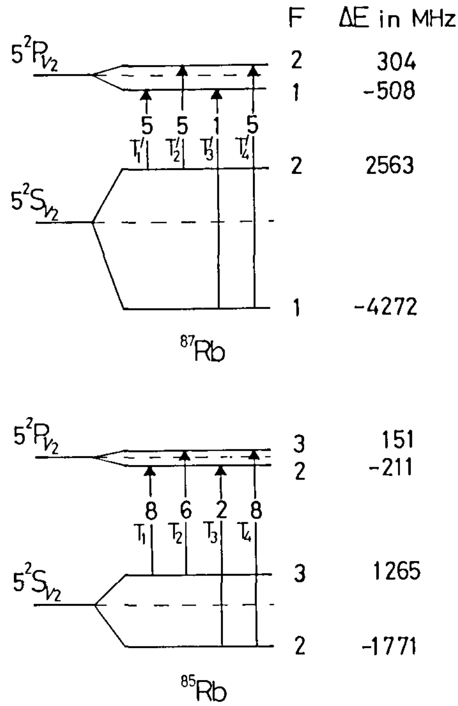

Fig. 2: Energy level diagram of Rb D_1 line. Possible transitions T_i for ^{85}Rb and T_i' for ^{87}Rb are indicated by arrows together with their relative transition probabilities.

III Experiments

Experiments were carried out by using a GaAlAs laser as a light source and the usual signal averaging technique. Figure 3 shows the block diagram of the experimental apparatus. A Rb cell of a 3cm inner diameter and 7 cm length was used and put in an electric oven at 435 K. The spatial temperature distribution was measured along the cell to be within 20 K. The vapor density was determined by the known vapor pressure data. The GaAlAs laser used was mounted on a copper block which was attached to a Peltier element driven by a temperature controller with high sensitibity. The frequency of the laser was tuned to the Rb D_1 line, by adjusting its temperature. To obtain the spectrum, the frequency of the laser was

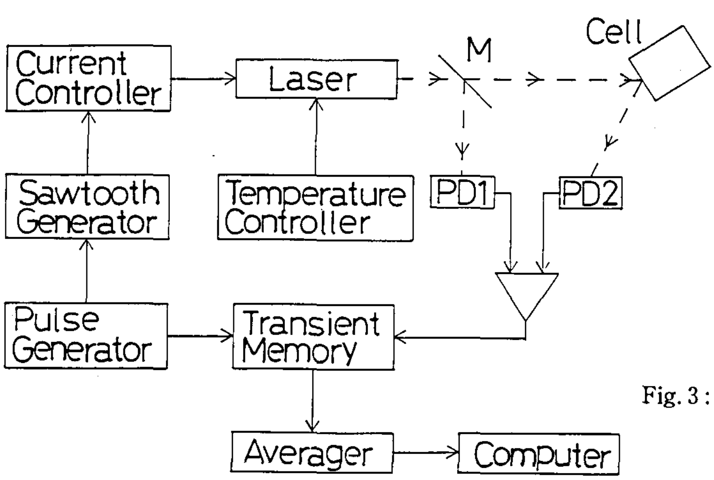

Fig. 3: Block diagram of the experimental apparatus.

scanned over about 10 GH^z, by sweeping the injection current of the laser diode. The light beam issued from the laser was split into two beams by a half mirror. One beam was used to monitor the intensity of the laser and also used as a reference voltage for compensation. Another beam with σ polarization was made incident on the glass-vapor interface, and the reflected light from the interface was monitored by another photodiode. The difference voltage between the outputs of the two photodiodes was amplified by an amplifier. The output voltage of the amplifier was averaged with a transient memory with averaging unit and sent to a personal computer for storage and recording of the signal.

Figure 4 (a) shows the theoretical positions and intensities of Rb D_1 line associated with transitions T_i and T_i'. Figures 4 (b) and (c) show the theoretical selective reflection spectra

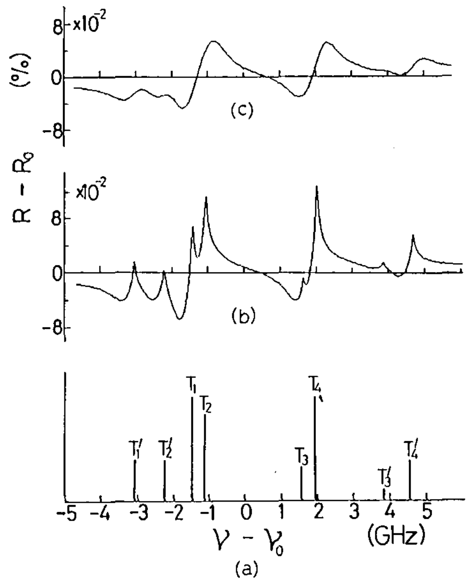

Fig. 4 : Selective reflection spectra of a glass-Rb vapor interface. Relative positions and intensities of transition T_i and T_i' are shown in (a). Theoretical selective reflection spectra calculated from the modified and the dispersion theories are shown in (b) and (c), respectively.

of Rb vapor calculated from the modified theory for the diffuse wall collision and the conventional theory for no wall collision, respectively. The former has been obtained from Eq. (18) for $\theta=\phi=0$. The latter has been calculated in terms of the expression which differs from Eq. (17) in that the factor 2 is absent and the integration runs over u from $-\infty$ to ∞[4]. The following data has been used in the calculation, taking into account of the experimental condition ; the Doppler width $\Delta\nu_D=610$ MHz at Rb vapor temperature of 435 K, the natural width $\Delta\nu_n=5.4$ MHz [7], the collision width $\Delta\nu_c=6.1$ MHz [7] where the vapor density has been calculated to be 1.8×10^{14} cm^{-3} from the experimentally known value of vapor pressure 8×10^{-3} Torr at 435 K[8] and the abundance ratio of ^{85}Rb to ^{87}Rb$=2.59$. It is found in Fig. 4(c) that the dispersion-shaped signals are obtained as four signals of ^{87}Rb for $T_i'(i=1-4)$ and two signals of ^{85}Rb, one for T_1 and T_2 and another for T_3 and T_4. The spectral width of each line is about 700 MHz which is approximately equal to the Doppler width 610 MHz calculated at 435 K. Narrowing of these signals, as seen on Fig. 4(b), occur with respect to the Doppler widths and there appear sharp peaks corresponding to T_i and T_i'.

Figure 5 (a) shows the theoretical selective reflection spectra of Rb vapor for various values of refraction angles θ's, which han been obtained from Eq. (18) by using the same data described above. Figure 5 (b) shows the corresponding experimental selective reflection spectra of Rb vapor at 435 K. The agreement between the theory and the experiment is satisfactory except that the base lines of the signals in Fig. 5(b) gradually shift downwards for increasing values of ω-ω_0. They are caused by changes of light intensities associated with modulated current which is added to the laser diode for the frequency sweep. If we subtract the above effect from signals in Fig. 5 (b), the line shapes and the signal intensities in Fig. 5 (b) are found to agree well with those in Fig. 5 (a).

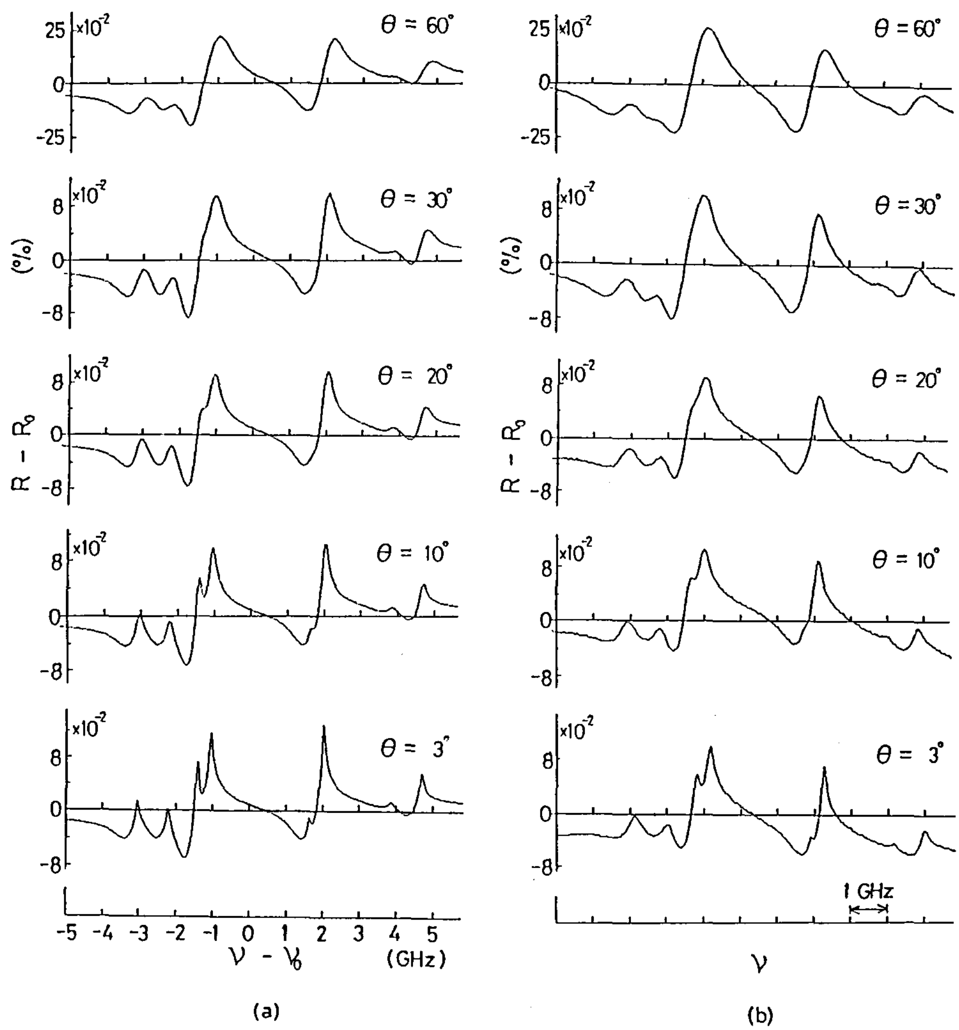

Fig. 5 : Selective reflection spectra of glass-Rb vapor interface for various refraction angles. Theoretical spectra for the modified theory and experimental ones are shown in (a) and (b), respectively. The spectra is represented by the extra reflectivity R-R_0 in the ordinate whose origin is taken as R_0 for a given θ.

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Rb 気体の選択反射信号の尖鋭化

伊 東 太 郎

Rb D_1 線の境界面からの選択反射スペクトルが入射光角度の関数として調べられた。スペクトルのドップラー広がりが、境界面への垂直入射光に対して部分的に減少し、ドップラー幅に埋没している ^{85}Rb の励起状態 $^2P_{1/2}$ の超微細構造が、このスペクトル尖鋭化の結果、ほとんど分解されるのが観測された。観測されたスペクトル線の形は原子と壁との衝突の効果を考慮した理論によるスペクトル線の形と良く一致している。